Uncertainty and Productivity: Exploring the Links

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Abstract

This note describes a framework to disentangle the sources of uncertainty facing firms and the channels through which uncertainty interacts with productivity. In particular, the note considers how a rise in potential productivity associated with technologies requiring intangible investment, can interact with business cycle fluctuations to generate persistent slumps, or uncertainty traps (**?**). The note also discusses slumps in innovative investment owing to uncertainty regarding the future direction of technology or to issues in the supply of financing for intangible investment. While much of the macroeconomic literature on uncertainty focuses on aggregate demand or supply shocks, recent work is exploring idiosyncratic uncertainty in business conditions that varies across heterogeneous firms or consumers. Finally, the note describes a heteregenous firm modelling, calibration, and simulation toolbox (under development) that will be part of the Micro Data Infrastructure.
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1 Introduction

Uncertainty is difficult to define with precision. Varying sources of uncertainty can affect economic decisions and the effect of uncertainty can vary by its source and by the affected economic decision maker. However, the qualitative economic effects of uncertainty are unambiguous, regardless of the source of uncertainty, its measurement, or the framework used for analysis. Uncertainty will delay irreversible decisions until information that becomes available during the delay resolves the uncertainty enough to enable a decision.[1] In this note we will catalogue sources of uncertainty that affect business decisions. We emphasize those sources of uncertainty that are relevant for economic policy, either because policy is the source of uncertainty, or because policy can resolve the uncertainty or reduce the economic impact of uncertainty.

Productivity is generally a measure of our ignorance; it is the source of increases in economic output when increases of all economic inputs are taken into account **?**. The theory and measurement of productivity varies, depending on the unit of analysis—firm, industry, country—the time period and frequency considered, and on the methodology used. Despite the variety of measures, productivity is a simple concept. At the micro level, productivity measures a firms' ability to generate output with economic inputs. At the aggregate level productivity is, perhaps more fundamentally, society's knowledge of how to generate well-being given resource constraints **?**. In this note, we extend a workhorse model of dynamics of heterogeneous firms (Hopenhayn **?**) that allows studying firm-level and aggregate productivity in an environment with uncertainty.

The concepts of uncertainty and productivity both have spawned a large literature. Relatively little has been written about their intersection.[2] In this note, we explore the links between uncertainty and productivity. The links can be causal in both directions. The main channel goes from uncertainty to investment. In this direction, uncertainty

[1] "Uncertainty about the long-run environment which is potentially resolvable over time thus exerts a depressing effect on current levels of investment **?**, p.63

[2] Of course, much macro literature uses 'productivity fluctuations' as a source of business cycles and stochastic volatility of the productivity process as source of uncertainty. In this note, aggregate productivity is the result of technology at the firm level interacted with various shocks at the micro, market, or aggregate level.

affects productivity through reductions in capital investment, which in turn lowers the trajectory of labor productivity. Uncertainty pertaining to aggregate demand or aggregate supply (e.g. import or energy prices, or framework conditions) may cause businesses to delay investing until they learn more about future business conditions. Models of this type can generate persistent slumps or uncertainty traps (Basu and Bundick **?** or Fajgelbaum et al. **?**). More directly, investment of a firm can be delayed owing to uncertainty in any factor relevant to that firm's 'business conditions'. Investment in innovation, R&D, or other intangible assets also can be delayed by uncertainty in business conditions, including uncertainty specific to the sector, or to the specific domain of innovation. Over time, this will cause the level of total factor productivity (TFP) to be lower, ceteris paribus, possibly exacerbating the uncertainty about the future.

The causal link from productivity to uncertainty is more ambiguous. In much of macroeconomic research, especially in the DSGE traditional (cite), the term 'productivity' is used as a shorthand for shocks to aggregate supply. Under this definition of productivity, it is productivity that is the causal source of uncertainty. Papers in this tradition will be considered in the group above, where uncertainty in aggregate demand or supply (other than productivity) causally affects productivity. Instead, we will discuss channels from firm-level productivity to macro uncertainty, where productivity, as defined by a shifter to production technology of a producing unit (firm), can alter (the precision of) expectations concerning aggregate developments. A traditional route is, for example, technological breakthroughs that change beliefs about the distribution of productivity, lead to investment and thus change expectations of aggregate conditions.

In this note we explore the possibility of productivity shocks affecting uncertainty. In particular, we consider the case where improvements in technological opportunities available to firms are associated with a necessary shift from tangible to intangible investment and a further skewing of firm outcomes. However, this shift in the make-up of investment results in a higher probability of uncertainty traps, with the technological boom period being misperceived as a macroeconomic slump. Uncertainty traps occur when the signals about aggregate volatility that firms receive from producing firms become less precise.

With a widening of the distribution of intangible-investment-based productivity draws and a further skewing of firm outcomes, the ratio of poorly measured intangible investment to total investment increases and the signal on macro fundamentals becomes mroe imprecise. It is thus the change in potential productivity that makes expectations about the volatility of future business conditions more imprecise. As a result, breakthrough technology can lead to perceptions of a poor macro fundamental, and thus to low investment, low adoption of the new technology, and low economic activity and aggregate productivity.

The key intervening variable in the interplay between productivity and uncertainty is 'business conditions'. As described by Bloom **?**, business conditions summarize the state of the world so that "Firms only hire and invest when business conditions are sufficiently good and only fire and disinvest when they are sufficiently bad. When uncertainty is higher...firms become more cautious in responding to business conditions." In focusing on business conditions, we limit our discussion of the effects of uncertainty on household decisions, e.g. saving or consumer durable expenditures, except through their indirect impact on business conditions, for example aggregate prices or interest rates.[3] In this note, we distinguish between dispersion (cross-sectional variance), volatility (time-series variance), and dispersion in volatility in the components of business conditions as a way to identify the role of productivity and uncertainty, partly following the work of Kozeniauskas et al. **?**.

Models of dynamics of heterogeneous firms with rudimentary general equilibrium (macro) features allow exploring the links between uncertainty and productivity. In the literature many extensions of such models allow rich analysis of innovation, trade, credit market conditions, labor market imperfections, etc. The model used in this note is a version of Hopenhayn **?** with production using capital and labor under decreasing returns and fixed operating costs. The model features free entry upon paying a fixed costs—that can be interpreted as an intangible investment—after which a persistent id-

[3]This partition becomes slightly blurred when some source of uncertainty affects both demand for a subset of goods and supply conditions of a subset of producers. We resolve this by understanding that business conditions could include idiosyncratic 'taste' shifters, as would be the case when deriving a firm's profit function from underlying utility and production functions.

iosyncratic productivity level is drawn. In this note, the main innovation is to provide a way to incorporate firm-specific, sectoral, and macro shock processes for both production and demand into a tractable, Markovian, state-space of business conditions. [4]. The model is calibrated using moments from micro-aggregated firm-level panels (ESSLait and CompNet-V8 data), and simulated to explore impacts of changes in uncertainty or of underlying firm technology.

We start the note by reviewing the relevant literature on uncertainty and productivity and by cataloging potential sources of uncertainty and possible ways to find empirical proxies. Next, we lay out the modelling framework and the calibration exercises. We use simulation to explore the possibility of uncertainty traps à la Fajgelbaum caused by a shift to intangible-based technology. We continue by describing the current state of a toolbox for studying firm dynamics that will be part of the Micro Data Infrastructure. We conclude with a discussion of the results and a map for future research.

2 Literature Review

Business conditions, as defined by Bloom et al. **?**, summarise the state of the environment which, usually in conjunction with the current capital stock, form the fundamentals for business decisions. In most models, business conditions evolve after production decisions are made. In order to make (partly) irreversible decisions, firms must therefore make 'forecasts' or form expectations of future conditions, properly weighting for probabilities of possible future outcomes. ".... But, as we have seen, the basis for such expectations is very precarious. Being based on shifting and unreliable evidence, they are subject to sudden and violent changes."(Keynes, The General Theory, Ch. 22., as cited by Bernanke **?** Nonetheless, progress has been made in finding measures that aid in forming expectation, but also in finding measures that measure the 'uncertainty' involved in expectation formation.[5] Proxies for uncertainty have included volatility of stock prices, disagreement

[4]The extension to demand shocks, and calibration of the parameters for the underlying processes is still under development

[5]Some scholars debate the possibility of resolving Keynes' quandary by assuming that uncertainty only is a question of temporarily incomplete information, e.g. **?**. Dow defines ambiguity as an assumption that quantitative cardinal probabilities are unknown but in principle knowable. Fundamental uncertainty

among forecasters, observed dispersion in firm-level outcomes, surveys among decision makers. Finally, while more difficult to quantify, narratives may provide a way to distinguish more clearly between underlying causes of uncertainty.

2.1 Sources of Uncertainty

Episodes of uncertainty ebb and flow, but currently macroeconomic uncertainty is likely high owing to the war in Ukraine, geo-political turmoil, commodity supply issues, inflationary pressures, and the climate transition, among others. In the EU, business confidence (EU Economic sentiment indicator) has gone below its long run average level in July 2022. Further, the EU consumer confidence indicator is at its lowest level ever in July 2022 (see **?**). The overall Economic Uncertainty Indicator (EUI) has been climbing back up towards its last peak reached at the onset of the Covid crisis in the past months.

In the literature, we have come across a wide set of proxies for economic uncertainty and economic policy uncertainty. To start, the VIX index, the Chicago Board Options Exchange's Volatility Index, measures volatility of stock prices based on S&P 500 index options (**? ?**). The VIX was historically high at the onset of the financial crisis and spiked at the beginning of the Covid-19 pandemic. Since then, the VIX has retreated, but to higher levels than following the financial crisis, and appears to be trending upwards. Bloom (2009) shows that the VIX is correlated with dispersion in growth of indicators such as profits or stock returns across firms. Further, all these indicators move countercyclically.

Unfortunately, observed dispersion from firm-level data does not track underlying shocks, but instead the effects of the shocks on firm decisions and market responses. In this sense, the firm-level uncertainty proxies are 'endogenous' and this endogeneity must be taken into account to filter out the underlying shock processes (e.g. **?**). In Bartelsman et al.**?**, it is argued that the observed cross-section dispersion in firm-level productivity reflects dispersion generated by firms investing in new, risky, technology, and 'destruction' by market selection of firms. By use of a model and indicators of

of Keynesian nature pertains to absence of quantifiable probabilities.

the productivity threshold for continuing firms, the underlying dispersion of the risky technology process can be retrieved.

The other indicator shown by Bloom (2009) to be correlated with the VIX is disagreement among professional forecasters. Here, the disagreement itself could be a proxy for the information content available in recent macro indicators for forecasting those indicators. More recently, **?**, **?** and **?** look at forecasts done by firm managers. Here, it is seen that firms' forecasts are predictive of their realizations on employment and investment, and that firms with lower forecast errors have higher profitability and productivity. Senga **?** finds significant heterogeneity in the uncertainty that firms face, and finds that firms have episodes where circumstance change so substantially, that they need to relearn making forecasts. Overall, this literature points to the fact that uncertainty of the macroeconomic environment is but one type of uncertainty that could impact firm's decisions.

Indeed, the European Investment Bank firm-level survey asks firms whether they have been held up in making investment decisions owing to uncertainty, and show that there is large variation across firms, but also that the proportion of firms answering in the affirmative does vary counter-cyclically. In Bartelsman, Chen, Kolev **?**, firm responses to the EIB investment survey are linked to their income statements from Orbis in order to parse out possible reasons why firms may state that uncertainty is holding back their investment. In the note, various underlying 'causes' for the uncertainty are discussed, with an assessment of their importance by looking at correlations between the survey response to the uncertainty question and proxy measures of source of uncertainty. Possibilities considered include proxies for firm-level innovativeness, flexibility for reallocation of resources across firms in a market, and globalisation. Also, geographic aspects are explored, indeed showing differences across countries in uncertainty as perceived barrier to firms' investment.

The exercise with the EIB firm-level survey can be thought of as providing support for a 'narrative approach' to uncertainty. Sometimes, firms worry about circumstances in local markets, sometimes they worry about the effects of Brexit, sometimes they worry about access to financing or qualified labor, and they always worry about their competitors. If any of these worries are related to uncertainty and result in postponing decisions, then the firm would answer affirmatavely in the survey. A correlation between the answers across firms and the more objective measure in the of the source of worry could be considered supportive of that narrative.

Which type of indicator one would like to use in empirical work depends of course on the question under study. In Kahn and Thomas **?**, **?** or Alfaro et al. **?**, the question relates to uncertainty in financial markets, in Altig et al. **?** it is about the Covid-19 pandemic, in Anayi et al. **?** about the war in Ukraine, Baker et al. **?** about uncertainty in economic policy, and **?** and **?** are concerned with uncertainty and business cycles in general.

2.2 Sources of Productivity

Before turning to measurement of productivity, we again start with the concept of business conditions. Business conditions summarize the the state of information, assets, and technology available to the firm when it makes decisions. Typical in production models, the state of the economy includes total factor productivity (TFP)—a shifter of the production function—as well as the stock of capital. Combined with knowledge of input prices, output prices or demand conditions, and the evolution of TFP, aggregate demand, and possibly idiosyncratic demand, firms make decision to enter a market, hire inputs, invest in capital, invest in innovation or new technology, etc. Many of these decisions are partly irreversible, and some, like decisions to innovate have the potential to change TFP. However, measured TFP also can change for reasons other than explicit innovative activity, adoption of technology, or learning activity. While these changes in measured TFP indeed occur, it is not evident why those movements would be causal for business cycle. On the contrary, these changes in measured TFP likely are caused by other underlying shock processes.

There is a long, wide, and deep literature on issues in productivity measurement. For firm-level productivity, reviews by Bartelsman and Doms **?** and **?** provide good overviews.

For macro and aggregate productivity measurement, O'Mahony et al. contains much information. For this note, the important issue is that typical measures of productivity are not directly the theoretical concept of the relation between change in output minus changes in input in which one is interested.

At the micro level, many issues make measurement of productivity difficult. To start, the change in inputs is endogenous to factors that may also change output, thus deeming their relationship unusable to measure productivity. Next, measured productivity could deviate from the underlying position of the production function owing to inaccurate or missing price information, changes in factor utilization, imperfection in credit or labor markets, frictions and adjustment costs, price-cost markups, (international) outsourcing, as well as general mismeasurement of output and input variables. Approaches to correct for some of these problems could for example be found in Olley and Pakes, or De Loecker (2011).

At the sectoral and macro level, some of the same issues occur. For measurement error in inputs and outputs, much effort is spent on price measurement and quality adjustment. Similarly, productivity measures by Fernald **?**, are meant to correct for capacity utilization. Nonetheless, issues in flexibility of resources and resource (mis)allocation, and discrete changes through firm entry and exit, could result in sectoral or macro productivity not reflecting the actual changes in output occurring through changes in inputs, ceteris paribus. While the use of time series information on measured TFP as a source of shocks or uncertainty may be

2.3 Uncertainty and Productivity

In this note, the questions of interest relate to productivity and possible (causal) links from productivity to uncertainty. Can new availability of intangible-based production technology cause the economy to enter an 'uncertainty trap'? Is investment in intangible assets more affected by macro uncertainty than investment in tangibles? Can uncertainty about societal acceptation of specific technologies slow down innovation (and shift to green technologies?)

Bontempi ? uses firm-level information on tangible investment and R&D spending to show how R&D flows are hampered by uncertainty. The work uses a reduced form model, which makes it difficult to provide an unambiguously causal interpretation of the results. David et al. ? explore the effect of uncertainty (imperfect information) on resource allocation in a dynamic firm model, and show how the uncertainty reduces aggregate productivity through misallocation. In Bonciani and Oh ?, uncertainty leads to a drop in aggregate demand. Their evidence is based on time series vector auto regressions and show that periods of high uncertainty (using uncertainty measure of ?) are followed by periods of low R&D and subsequently low TFP.

The above papers look at investment following a period of increased macroeconomic uncertainty. However, there are more future conditions that are relevant to a firm making (partly) irreversible decisions. A particularly lumpy decision is starting a new business. If entry also entails investment in intangible assets with uncertain technological and market payoffs, then the decision can be postponed while uncertainty resolves. In particular, if one is in a period of rapid technological change, entrepreneurs have to weigh technological risk as well as market risk. Similar to Acemoglu ?, benefits of investments in new technology relate to the reduction in costs or additions to consumer benefits, per unit, times the scale of the market. While entrepreneurs and innovators are specialists in weighing the technological risk, the scale of the future market often is beyond their control. For example, innovation in zero-carbon transportation depends on future regulatory developments and oil prices, but also on paths for future infrastructure, e.g. hydrogen vs electric fueling stations. In exploring the links between uncertainty and productivity, it is therefore important to include macroeconomic (aggregate demand) conditions, (sectoral) demand conditions, aggregate technological opportunities, as well as idiosyncratic firm-level productivity shocks.

Of special interest is the model of uncertainty traps of Fajgelbaum et al. ?, where firms need to learn about the state of aggregate demand through a noisy signal. In this note, we borrow the learning mechanism from Fajgelbaum et al. so that positive news of new potential intangible-based technology can end up reducing the precision in the learning process that is needed to recover from a deep recession.

In the Fajgelbaum paper, the uncertainty trap mechanism is first explained in a simplified model with a fixed number of risk averse firms whose fundamental output is drawn from an autoregressive shock process with given, but unknown, variance and zero mean. To be eligible for receiving output (the fundamental of the economy), firms pay a fixed cost (investment) that is drawn from a distribution with a known mean and variance. Given the fixed cost and the firm's belief about the value of the fundamental, it decides to either wait for more information to update its beliefs, or to invest. All information is public, so all firms have the same common beliefs. Firms base their beliefs about the fundamental (its mean and variance) on a noisy public signal about the fundamental and from a signal generated by operating firms, that is firms that invest. If firms think that fundamentals are bad, then fewer firms will invest. But, if fewer firms invest and the ratio of poorly measured intangible to tangible investment increases, then the signal about the fundamentals generated by firms will be noisier. Noisier signals will make it harder for firms to update their estimate of the variance of the fundamental process. A higher number of operating firms and more precisely measured intangible investment will more quickly reduce uncertainty.

In this framework, a self-fulfilling expectational 'trap' can cause the economy to remain in a low investment environment for prolonged periods. In our model, we allow the possibility that positive technology shocks, especially those that substitute for capital, lead to an incorrect inference that aggregate demand is low. The paradoxical result is that positive news on technology can lead to persistently low investment as well as a skewed distribution of technological winners and laggards.

3 Model

In this section, we present a generic heterogeneous firm dynamics model, based on **?**. Some of the model parameters are calibrated using firm-level micro moments, and simulated to understand the possibility of uncertainty traps originating from new potential technology. The modelling framework, however, is more generic and forms the basis of a toolbox to be added to the Micro-data infrastructure.[6]

The modelled economy is populated by incumbent firms with heterogeneous productivity and an unlimited mass of potential entrants. Firms face idiosyncratic productivity shocks but also sectoral and macro shocks. The uncertainty regarding the shocks can vary over time. Firms choose to enter or continue, and if they continue, they optimally decide on hiring and tangible capital investment. Entering firms must pay an entry fee that can be interpreted as an investment in intangible assets. Upon paying the fee, they get a persistent draw from a (known) productivity distribution. Further, firms face some idiosyncratic productivity shocks. Market uncertainty also can appear in the form of macro demand shocks or idiosyncratic (sectoral) demand shocks.

3.1 Supply-side environment

Incumbent firms, who face business conditions B, can produce output Y using capital, K, and labor L with the following production technology:

$$Y_{it} = B_{it}(L_{it}^{\alpha} K_{it}^{1-\alpha})^{\gamma} \tag{1}$$

where $0 < \gamma \leq 1$ and $0 < \alpha < 1$. Henceforth, we leave out i and/or t subscripts when they are unambiguous. The capital stock evolves through investment I and is subject to depreciation δ such that $K_{t+1} = (1-\delta)K_t + I_t$. Business conditions partly reflect persistent firm-level productivity, A_i, that is drawn from a known distribution upon entry.

Incumbent firms make decisions on hiring, continuation, and investment. In order to produce, firms also need to pay a fixed operating cost, c_f. Besides an endogenous continuation decision, the model features an exogenous exit hazard, λ. In simplified versions of the model with no endogenous exit, this can be set to a rate that can match observed firm exit rates, but even with endogenous exit decisions, a small exogenous

[6] The micro-data infrastructure (MDI) is being developed under work package 1 of the EU Horizon2020 project Microprod. The modelling framework, simulation, and calibration tools being developped will work alongside the tools being built for research with linked firm-level datasets.

exit hazard allows for a steady state (ergodic) firm distributions in an environment with Markovian shock dynamics.

There is a continuum of potential entrants who pay an entry fee, c_e in order to receive a persistent draw, A_i. We interpret c_e to be the only path of intangible investment in the economy, with the combined entry fees paid by entrants providing payoffs from the stochastic A_i draws that provide profits to producing firms. There is a free entry condition that ensures that firms are indifferent at the margin, or in other words that the value of a new firm upon entry equals the entry fee. A key to the simulation of uncertainty traps is changing c_e and the distribution of A_i in a way to mimic new intangible-based technology becoming available.

Besides persistent productivity shocks, firms face time-varying idiosyncratic shocks to productivity, z_{it}, drawn from a known distribution. These shocks are part of the state of business conditions. Consistent with evidence, the timeseries volatility of within-firm productivity is much lower than the cross-sectional dispersion. Further, since evidence on the volatility is endogenous, careful calibration must be used to disentangle the time-varying idiosyncratic productivity shocks from demand-side shocks.

The timing in this discrete time model is as follows:

- At the start of period t, Incumbents (continuers from the previous period plus continuing entrants from the previous period) pay the fixed fee c_f, optimally hire labor L, and produce with the current state of business conditions B_t and the beginning of period capital stock, K_t.
- New entrants pay the entry fee c_e and receive the initial productivity draw A_i.
- Incumbents and new entrants form expectations about future business conditions and decide whether to continue.
- Conditional on continuation, firms invest $I_t > 0$ to achieve their desired stock of capital K_{t+1}
- Continuers (and continuing entrants) receive shocks to business conditions, conditional on the current state of each of the shocks. Also, exogenous exit occurs at

rate $\lambda > 0$.

- period t ends.

3.2 Business Conditions

In this note, we emphasize the notion of 'business conditions' as describing the state of the environment that is relevant for decisions by heterogeneous firms.

On the firm side, we have already discussed the persistent productivity draw, A_i and the idiosyncratic productivity shock, z_{it}. If we assume that firms face some downward sloping demand curve of the form $Y_{it}^d = X_s P_{it}^{-\epsilon} P_Q^{\epsilon-1} Q_t$, with X_s being an idiosyncratic taste shifter for goods from a certain sector, and Q_t reflecting aggregate demand. Overall, if firms set output such that marginal revenue equals marginal cost and choose inputs to maximize profits, we end up with $y_{it}^* = B_{it} F(w_t, r_t | \gamma, \epsilon, \alpha)$, with the partial derivative w.r.t. input prices being negative. The expression for B as a function of the underlying components of business conditions and the model parameters is then:

$$B_{it} = (A_i z_{it})^{\frac{1}{1-\theta\gamma}} (\frac{1}{\theta}\frac{1}{\gamma} X_s Q_t)^{\frac{\gamma}{\theta\gamma-1}} \tag{2}$$

where $\theta = 1 - \frac{1}{\epsilon}$, and under the assumptions that firms are under monopolistic competition and set output such that marginal revenues equal marginal costs and have made value maximizing decisions.

For each of the underlying shock processes, we discretise the distribution and generate a Markovian transition matrix that gives for each grid position (row) the probability of moving to another grid position (column). The probabilities sum to one. The transition matrix for B, Pr_B, is given as the Kronecker product of the transition matrices of the individual shock processes. The dimensionality of the overall transition matrix is thus the product of the dimensions for each separate discretised distributional grid.[7]

[7] For typical AR(1) or n-state transition matrices, the grid is based on equal distances of the support of the distribution. For the initial draw, A_i, the grid is based on quantiles, and the value in the grid is the mean of the integral of the density times the quantile value. This later choice is made because the grid boundaries at lower values of the A draw would be far apart, just in the area where a fine grid is needed to discern more precisely the exit, or non-continuation value of firms.

3.3 Firm Decisions

In the economy, labor is supplied according to $L^s = \bar{L}w^\eta$, where the market wage, w, is determined such that aggregate optimal labor demand equals labor supply, L^s .

Taken together, the short-run profit, assuming flexible capital, is given by:

$$\Pi((B_{it}|A, z, X, Q), K, L|w) = (X_s Q_t)^{1-\theta}(A_i z_{it})^\theta L^a K^b - w(L + c_f) - \rho K \tag{3}$$

where $a = \alpha\gamma\theta$ and $b = (1-\alpha)\gamma\theta$. Instantaneous optimal labor demand is then $L^* = \frac{a}{w}(X_s Q_t)^{1-\theta}(A_{it} z_{it})^\theta K^b$. The concentrated profit function can then be written as

$$\Pi(B, K) = a^a B K^{(1+a)b} - c_f - \rho K \tag{4}$$

where $B = (X_s Q_t)^{(1-\theta)(1+a)}(A_i z_{it})^{\theta(1+a)}$ is the relevant state of 'business conditions'.[8] We assume in the baseline that the user cost of capital $\rho = \bar{r} + \delta$ is constant and given exogenously, implicitly assuming a global supply of financial capital. Extensions are to embed this model into a macro framework with households making time-varying consumption/savings decisions by optimizing a given utility function.[9]

The firm must decide, given knowledge of the shocks A_i, z_{it}, X_s, Q_t, the underlying stochastic parameters, and expectations of their Markovian evolution, whether to continue in the next period, and if so, how much to optimally invest for desired K_{t+1}. Formally, firms continue when their value is positive:

$$V(B, K) = \text{Max}_{\mathbf{C},I}(\Pi(B, K) - I + \mathcal{I}_{\mathbf{C}=1}\beta(1-\lambda)E(B'|B(A, z, X)V(B', (1-\delta)K + I) \tag{5}$$

where the indicator for continuers, $\mathcal{I}_{\mathbf{C}=1} = 1$ when the expected future value is non-negative, zero otherwise and β is the exogenous discount factor. The expectations, $E(B'|B(A, z, X)$ (notated with $'$) of future business conditions are conditional on the

[8] For now, the textual display and the various choices that can be made in the modelling framework are not quite consistent. The baseline model assumes a single demand shock Q_t, and instantaneous adjustment of tangible capital.

[9] Further, future extensions are important to understand differing supply conditions for funding of tangible and intangible investment, see e.g. **?**.

individual underlying shocks of the current state of business conditions.[10] In this setting, the value function splits into the value of continuing and investing, or continuing and not investing, i.e. letting depreciation decay the stock of capital

$$V_{C=1}(B,K) = \max[V^I(B',(1-\delta)K+I), V^W(B',(1-\delta)K)] \tag{6}$$

The free entry condition states that entry will take place until the expected value of entrants equals the entry fee. Since entrants don't produce in the period they enter, the value at each draw of A_i is the discounted—by β and by then chance of not hitting an exogenous exit, $1-\lambda$—expected value of the business conditions they have could have in the following period, taking into account the transition probabilities. In this calculation, there will be entrants who receive a draw of A_i under the continuation threshold, return a value of zero.

3.4 Market Equilibrium

A steady-state market equilibrium is given by: aggregate demand equals total supply of incumbents, aggregate labor supply equals total demand for labor of incumbents, the expected value of an entrant equals the entry fee, and 'state of the industry' is in steady-state: the distribution of firms by B is at a fixed points, where the endogenous and exogenous exiters are replaced by entrants, and aggregate gross investment equals depreciation plus capital of exiting firms. For every value of business conditions in the state space, the equilibrium provides us with policy functions for incumbents decisions on continuation, labor input, and investment. Further, we know the steady state wage (and output price, in case of elastic demand). For simulations, we can assess what happens to firm's choices and aggregate outcomes when business conditions fluctuate. Mechanically, (improperly measured) productivity, aggregate output over aggregate capital and labor, will fluctuate procyclically as the utilization of intangible assets changes. More interestingly, we can see what happens to tangible and intangible investment and to actual

[10] In the next section, we will describe more fully the construction of expected business conditions from the underlying 'memoryless' stochastic processes.

productivity.[11]

3.5 Calibration

The baseline version of the model has about two dozen parameters. For each of the shock processes, we need a functional form or dynamic equation and some distributional parameters (mean and variance for 2-parameter families). For the production model, we require the labour output elasticity, returns to scale parameter, overhead labor, entry fee and a parameter for the 'scale' of the economy (e.g. 'natural' labor supply. We further need a capital depreciation rate, an exogenous (or trickle) exit rate, and an interest rate. Finally, some parameters technical parameters, such as number of grid points and convergance tolerances need to be set.

For this note, we have access to various databases with firm-level moments. We choose first to calibrate the baseline model using data moments from before the financial crisis, smartphones, and AI technologies. The ESSLAIT project provided moments from linked firm-level panels for 14 EU countries.[12]. As a first exercise, we choose the German non-high-tech manufacturing sector (EUKLEMS acronym MexElec) for the year 2008, in order to collect data from a large sample of firms in a period and sector with 'traditional' manufacturing technology. At a later date, we will compare the baseline results to the high-tech manufacturing in a country with high rates of intangible investment. Using a public-use version of the ESSLait data,[13] we have total employment, number of firms, nominal value added, wage, capital stock, sectoral labor productivity and TFP, and distributional indicators, such as mean labor productivity by productivity quartile and the share of employment by firm size class. All these indicators can be computed as model-moments from simulations.

Some parameters can easily be calibrated 'from the literature', e.g. the interest rate) (4pct, the depreciation rate (8pct) and the labor share (2/3). From earlier firm level

[11] A difficulty, remaining to be solved, is the adjustment for the entry margin: does incipient entry under good business conditions raise the entry rate and lower incumbents values, or does it increase the wages resulting in a decrease in all firms values.

[12] ESSNET Grant agreements number 50701.2010.001-2010.578 and 49102.2005.017-2006.128, `https://ec.europa.eu/eurostat/cros/content/esslait_en`.

[13] `https://ec.europa.eu/eurostat/web/digital-economy-and-society/methodology`

work ?, we pin the overhead fixed cost at 0.14. In case we only allow exogenous exit, the exit rate is set to mimic firm-level exit (around 6pct), otherwise it is set at 0.1pct. The 'natural' employment, $\bar{L}$ is directly picked from the ESSLait data.

The calibration next consists of searching over the space of parameters of interest to minimize a loss function of the sum of squared (log) differences between data moments and model moments. The calibration is done for simulations of the steady state model. The parameters we chose for the calibration exercise are the fixed entry cost, the returns to scale parameter, and the shape and scale parameters of the gamma distribution for A_i.

In later work, we will run the calibration on CompNet data for Germany in 2018, for both the hightech and non-high tech manufacturing sectors, and rerun the 2008 calibration for the high-tech manufacturing as well.

Finally, the shock process needs calibration. The macro shock can be derived from DSGE literature, while the idiosyncratic, time-varying firm productivity shocks will need information on distributions of productivity growth rates. (to be done).

Overall, a preliminary calibration allows us to choose $\bar{L} = 3.9$ million, $c_e = 50$, $\gamma = .87$, and the mean of the gamma distribution at 30 and the scale=1 (given mean=30 (thousand), and variance 30).

The technical parameters for steady state calibration are set: 60 grid points for A_i and tolerance for iterative solutions at $1e^{-5}$.

3.6 Simulation for Uncertainty Traps

In the simulations, we will increase the mean and variance of the initial distribution, A_i, and increase the entry fee in a way that mimics the availability of new intangible technology, but does not change steady state TFP. In steady state, these changes will increase the ratio of intangible to tangible investment and will decrease the entry rate.

Both of these changes, when embedded in the Fajgelbaum et al. model, will make it more difficult for firms to learn about the state of macro shocks because the information content of the signal becomes smaller. Because of this, following large enough negative macro demand shock, firms will think they are in a bad state of the world, but owing to lower signal precision than before the technological change, they are more likely to remain in an 'uncertainty' trap.

3.7 Toolkit

The dynamic firm modelling toolkit will contain a set of modules for specifying, calibrating and simulating Hopenhayn-style (?) models.

For the production framework, we allow:

- Cobb-Douglas CRTS production with capital and labor embedded in a monopolistically competitive output market with a given demand elasticity. Other functional forms derived from more interesting demand systems than CES will be added in the future (see e.g. Atkeson and Burstein 2008).
- Cobb-Douglas with returns-to-scale parameter $\gamma < 1$. A combination of demand elasticity and returns to scale parameter needs to result in concave profit function in order to support a non-degenerate economy with heterogeneous firms (see e.g. Mrazova et al. ?).
- A combination of monopolistic competition and decreasing returns to capital and labor.

Further, the modelling framework allows specification of various types of shocks on the demand and production (supply) side. For each, a choice can be made for AR(1) process (with Rouwenhorst discretisation), a n-point Markov chain, or a static distribution with a 'trickle' decay, and a resulting transition matrix can be applied in the necessary value function iteration. Further, the variances of the underlying shocks can all be subject to stochastic volatility to simulate uncertainty shocks.[14] Because the transition probability for a particular value for overall 'business conditions', will depend on the composition of the underlying shocks and their transition probabilities, the state space will consist of

[14] For now, the model is simulated under different variance assumptions to mimic the improvements in potential intangible-based production.

the cartesian product of all the separate grid points. The curse of dimensionality rears its ugly head.

The economic environment can differ, with assumptions of a perfectly elastic labor supply, perfectly inelastic labor supply or a given positive elasticity; on the demand side: perfectly elastic product demand, perfectly inelastic demand, or a given negative demand elasticity, or a combination with both being elastic (perfectly elastic for both, or perfectly inelastic for both are ruled out). Further, policy can tinker with exit costs, entry costs, overhead costs, taxes and subsidies for inputs or outputs, and other such features.

Not yet included are richer specification of innovation, for example by adding horizontal or vertical product innovation.

For calibration, the Microprod team is working on building an interface between firm-level modules to collect moments that are useful for calibration of the model. Automating choices, for example definitions of quantiles, size classes, variable definitions, etc, could aid in matching moments from simulated model output with moments from firm-level data. At present, some rudimentary tools are available for searching the model parameter space to minimize loss function from deviations between data and target moments.

4 Conclusion

In this preliminary note, we describe the first results of research employing simulations of general equilibrium firm dynamics models that are calibrated to micro-moments data. The first exercise conducted makes the point that the phenomenon of 'Uncertainty Traps'—where a recession may turn into a prolonged slump because agents have difficulty getting precise estimates of the variance of macro shocks following a sharp negative shock—becomes more likely in a world where intangible-based technological opportunities arrive.

A new feature of this research is the rich specification of business conditions that enhances the policy relevance of heterogeneous firm dynamics models. In the modelling framework described in this note, modifications can be made to the particulars of business condition environment to study labor market frictions, financial frictions, sectoral policy, long-term innovation policy, trade policy etc. Such questions could be answered, one at a time, in a broad range of papers using customized heterogenous firm frameworks tailored to each area. However, having a coherent research environment adaptable to many areas of study, will both reduce costs for individual researchers and improve the comparability of findings across studies.[15].

[15]The ambition is to see the current note as a kickoff for an endeavour similar to Dynare, that enables research using common tools for DSGE and OLG models https://www.dynare.org/about/

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